Workshop
BILDVERARBEITUNG FÜR DIE MEDIZIN
Algorithmen - Systeme - Anwendungen
Lübeck 2015
15.- 17. März

Tutorial 2: Medizinische Bildverarbeitung

Prof. Dr. J. Modersitzki

J. Lotz, Dr. N. Papenberg, M. Schenk, N. Traulsen

Fraunhofer MEVIS, Lübeck

Fraunhofer
MEVIS

Institute of
Mathematics and Image Computing
University of Lübeck, Germany

UNIVERSITÄT ZU LÜBECK
INSTITUTE OF MATHEMATICS
AND IMAGE COMPUTING

jan.modersitzki@mic.uni-luebeck.de
http://www.mevis.fraunhofer.de/
http://www.mic.uni-luebeck.de

Contents

- General introduction, iIntroduction to image registration
- Applications, examples, setup
- Variational approaches
- Data and transformation models
- Distance measures
- Parametric registration
- Regularizers
- Non-parametric registration
- Optimization
- Hands-On-Demos using FAIR Matlab toolbox





University of Lübeck

Part of BioMedTec Campus

- Fachhochschule Lübeck
- Fraunhofer EMB, Einrichtung für Maritime Biotechnologie
- Fraunhofer MEVIS, Lübeck
- Leibnitz Zentrum, Borstel
- UKSH – Universitätsklinikum Schleswig-Holstein
- University of Lübeck
  - Medicine: 39 Institutes
  - MINT: 15 Institutes
  - MINT: 8 Institutes
- Small, Excellent, Interactive, Multidisciplinary

MINT Section: Math, Info, Nat.-Sci, Tech

1. Entrepreneurship und Business Development
2. Informationssysteme
3. Mathematik
4. Mathematische Methoden der Bildverarbeitung
5. Medizinische Elektrotechnik
6. Medizinische Informatik
7. Medizintechnik
8. Multimediale und Interaktive Systeme
9. Neuro-/Bioinformatik
10. Robotik und Kognitive Systeme
11. Signalverarbeitung
12. Softwaretechnik und Programmiersprachen
13. Technische Informatik
14. Telematik
15. Theoretische Informatik

MIC: Mathematics and Image Computing

- Prof. Dr. B. Fischer, 1957–2013
- Prof. Dr. J. Modersitzki
- Prof. Dr. N.N.
- Dipl.-Math. Constantin Heck
- M.Sc. Thomas Polzin
- Dipl.-Math. Sebastian Suhr, with M. Burger, WWU Münster

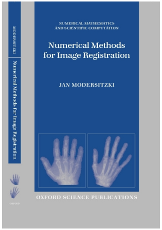
MIC: Mathematics and Image Computing

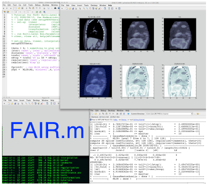
- Image Enhancement, Denoising
- Segmentation
- Registration
- Reconstruction

- Classical Methods (Filter, Fourier, Wavelets, . . .)
- Inverse Problems
- Modeling
- Numerical Methods for Image Processing
- Numerical Optimization
- Partial Differential Equations
- Variational Methods

MIC: Mathematics and Image Computing

Image Registration, Data Fusion, Motion Correction





- Jan Modersitzki, Numerical Methods for Image Registration, Oxford University Press, New York, 2004.
- FAIR: `http://www.siam.org/books/fa06/`
- Jan Modersitzki, Flexible Algorithms for Image Registration, SIAM, Philadelphia, 2009.

Joseph von Fraunhofer

1787–1826

Optics and Physics

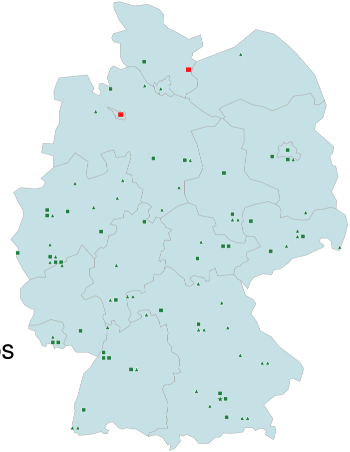
Fraunhofer Gesellschaft

- applied research & prototyping (car industry, MP3, etc)
- Bridges industry and academic
- Largest organization for applied research in Europe:
  ≈ 23.000 employees
  ≈ 2,1 billion € per year
- about 80 units

■ Institutes and ▲ Project Groups

■ Fraunhofer MEVIS, Bremen

■ Fraunhofer MEVIS, Lübeck



Fraunhofer MEVIS

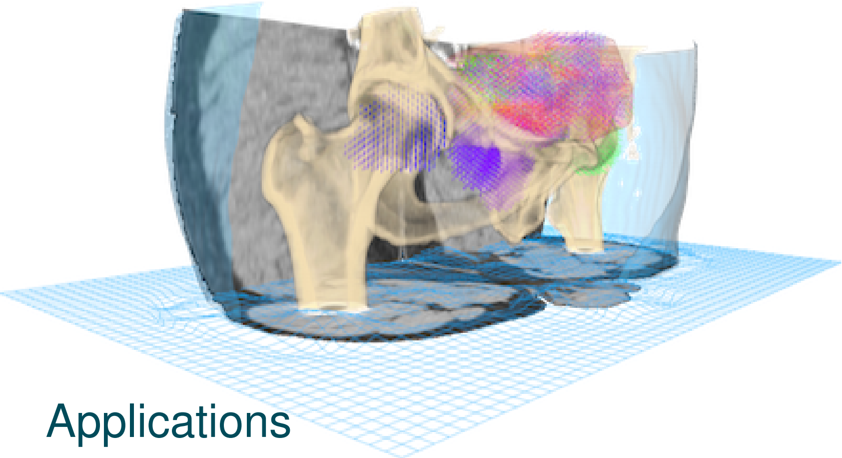
- Institute for Medical Image Computing

  sites at Bremen and Berlin, Heidelberg, Lübeck, Nijmegen
  ≈ 100 researcher
  $\approx 9 \cdot 10^6$ €

- Multidisciplinary R&D
  - Image acquisition, analysis, computing, denoising, enhancement, reconstruction, segmentation, visualization
  - Modelling and simulation
  - Applications, workflow and usability engineering

- Solutions for Clinical Problems

Fraunhofer MEVIS & MIC, Lübeck

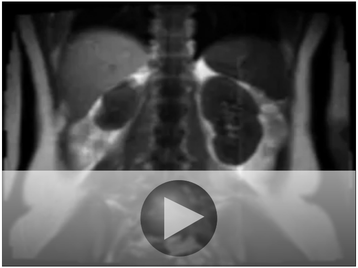


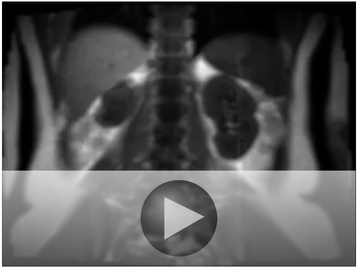
Applications

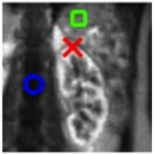
Dynamic Contrast Enhanced Magnetic Resonance Imaging

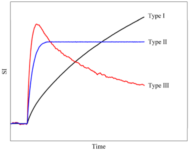
Constantin Heck

with J. Rørvik, Haukeland Clinic and A. Lundevold, U Bergen





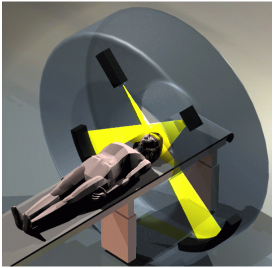


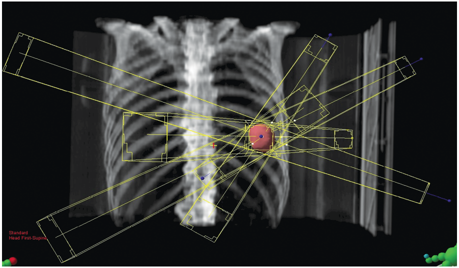


Ruthotto, Berkels, Heck, and Modersitzki: *Model-based parameter estimation in DCE-MRI without an arterial input function*, BVM 2014

Radiation Therapy

Mark Schenk, Nadine Traulsen
with Benjamin Haas and Michael Wachsbüsch, Varian Medical

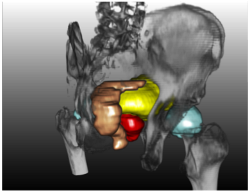




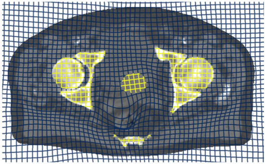
Local Rigidity in Radiation Therapy

Lars König, Nils Papenberg

with Benjamin Haas and Michael Wachsbüsch, Varian Medical



Deformation grid (■) with rigid areas (■)

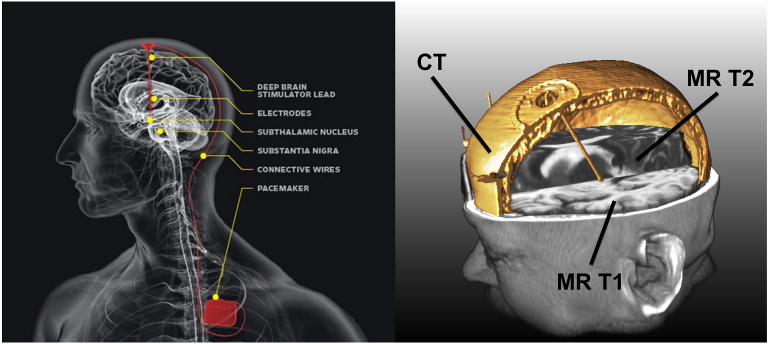


König, Papenberg, Haas, Modersitzki: *Deformable Registration for Adaptive Radiotherapy with Guaranteed Local Rigidity Constraints,* Varian Research Partnership Symposium 2013

Guided Intervention: Deep Brain Stimulation

Kanglin Chen, Stefan Heldmann, Jan Rühaak

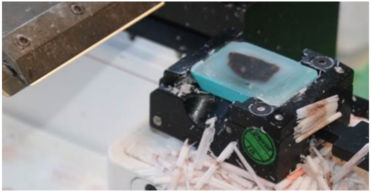
with Sapiens Steering Brain Stimulation B.V./NL

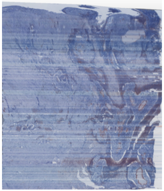


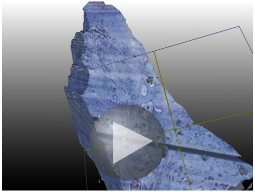
Rühaak et al.: *Accurate CT-MR image registration for deep brain stimulation: a multi-observer evaluation study,* SPIE 2015

Digital Pathology, Virtual Histology

Judith Lotz, Johannes Lotz, Janine Olesch, Herbert Thiele



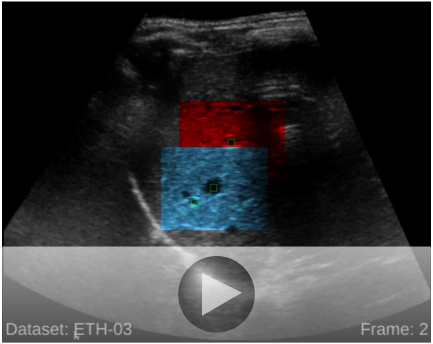




Lotz et al.: *Zooming in: High Resolution 3D Reconstruction of Differently Stained Histological Whole Slide Images*, SPIE 2014

Real-time Tracking

Till Kipshagen, Lars König, Jan Rühaak

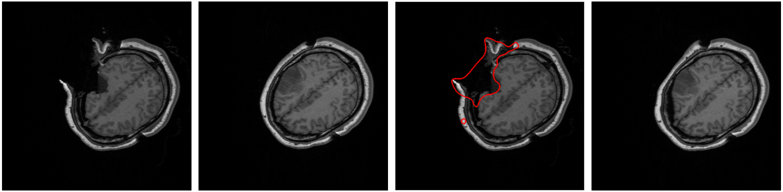


König, Kipshagen, Rühaak: *A Non-Linear Image Registration Scheme for Real-Time Liver Ultrasound Tracking using Normalized Gradient Fields,* MICCAI Challenge CLUST 2014

Automatic Detection of Non-Correspondences

Kanglin Chen, Alexander Derksen

Brain-Shift



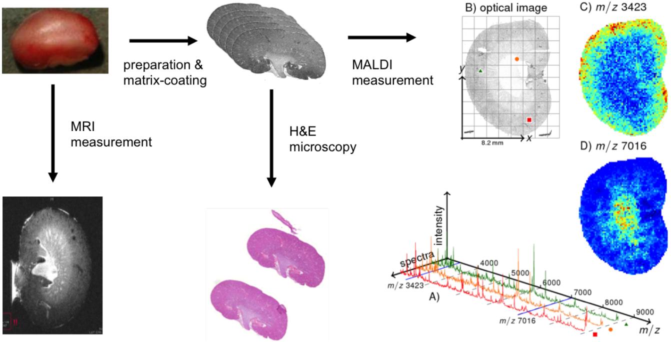
$$\mathcal{J}(y, \Sigma) = \int_{\Omega \setminus \Sigma} |\mathcal{T}[y] - R|^2 \, dx + \mathcal{S}[y] + \mathrm{Vol}(\Sigma) + \mathrm{Per}(\Sigma) \xrightarrow{y,\Sigma} \min$$

Chen, Derksen, Heldmann, Hallmann, Berkels: *Deformable Image Registration with Automatic Non-Correspondence Detection*, SSVM 2015

MALDI: Matrix-Assisted Laser Desorption/Ionization, 1 of 2

Stefan Heldmann, Judith Lotz, Herbert Thiele

BMBF funded

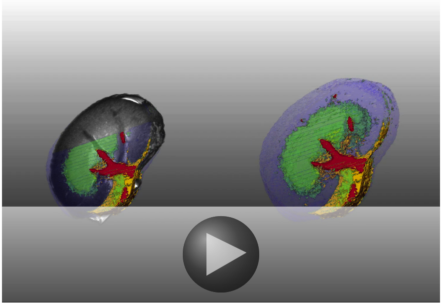


Thiele, Heldmann, Lotz: *2D and 3D MALDI-imaging: Conceptual strategies for visualization and data mining*, Biochem Biophys Acta, 2014

MALDI: Matrix-Assisted Laser Desorption/Ionization, 2 of 2

Stefan Heldmann, Judith Lotz, Herbert Thiele

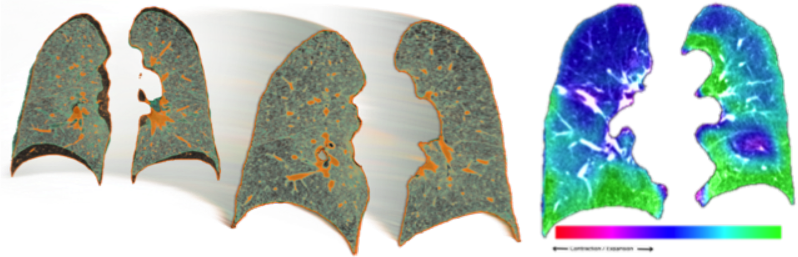
BMBF funded



Oetjen, et al.: *Three-Dimensional MALDI Imaging Mass Spectrometry using PAXgene Fixation,* Journal of Proteomics, 2013

Lung Registration with Volume Analysis

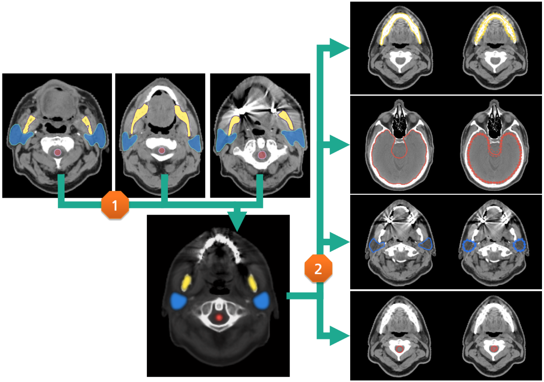
Stefan Heldmann, Till Kipshagen, Jan Rühaak
with Hoen-oh Shin from Hannover Medical School



Rühaak, Heldmann, Kipshagen, Fischer: *Highly Accurate Fast Lung CT Registration*, SPIE 2013

Atlases: Information Transfer

Kanglin Chen, Alexander Derksen, Marc Hallmann



Chen, Derksen, Hallmann: *A Variational Method for Constructing Unbiased Atlas with Probabilistic Label Maps*, BVM 2015

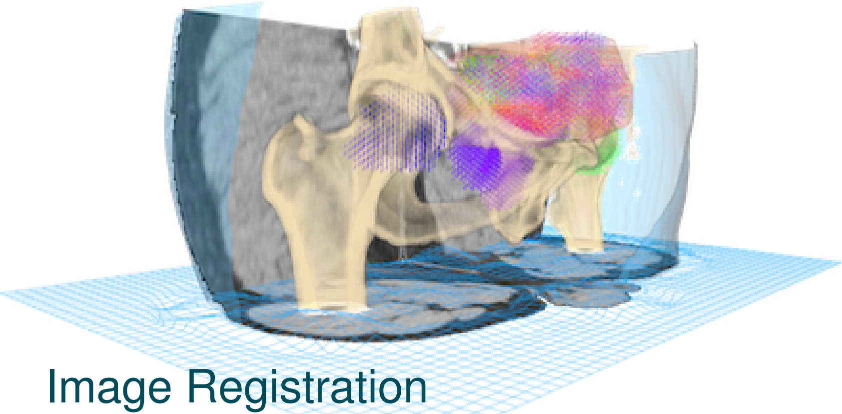


Image Registration

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \stackrel{y}{\longrightarrow} \min, \qquad y \in \mathcal{A}$$

Images Registration Approaches

- Demons
- Discrete approaches (optimization)
- Heuristics
- Mass-Transportation (Monge-Kantorovich Problem)
- Optical Flow
- Transport problems (geodesics, diffeomorphisms)
- Statistical approaches: Maximum A Posteriori probability
- Variational approaches

References, Books

- A. A. Goshtasby.
  2-D and 3-D Image Registration.
  Wiley Press, New York, 2005.
- J Hajnal, D Hawkes, and D Hill.
  Medical Image Registration.
  CRC Press, 2001.
- J. Modersitzki.
  Numerical Methods for Image Registration.
  Oxford University Press, New York, 2004.
- J. Modersitzki.
  FAIR: Flexible Algorithms for Image Registration.
  SIAM, Philadelphia, 2009.
- Terry S. Yoo.
  Insight into Images: Principles and Practice for Segmentation, Registration, and Image Analysis.
  AK Peters Ltd, 2004.

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M. Burger, J. Modersitzki, and D. Tenbrinck.
Mathematical methods in biomedical imaging.
In G Kutyniok, G Plonk-Hoch, and G Steidel, GAMM Mitteilungen, 2014.

J. M. Fitzpatrick, D. L. G. Hill, and C. R. Jr. Maurer.
Image registration.
In M. Sonka and J. M. Fitzpatrick, *Handbook of Medical Imaging: Medical Image Processing and Analysis*, SPIE, 2000.

D. L. G. Hill, P. G. Batchelor, M. Holden, and D. J. Hawkes.
Medical image registration.
Physics in Medicine and Biology, 46:R1–R45, 2001.

S. Heldmann, J. Modersitzki, and N. Papenberg.
Nonlinear registration via displacement fields.
In A W Toga, P Thompson, and K Friston, *Brain Mapping: Methods & Modelling*, 2014, to appear.

References, Articles II

- Hava Lester and Simon R. Arridge.
  A survey of hierarchical non-linear medical image registration.
  Pattern Recognition, 32:129–149, 1999.
- J. B. Antoine Maintz and Max A. Viergever.
  A survey of medical image registration.
  Medical Image Analysis, 2(1):1–36, 1998.
- L. Ruthotto and J. Modersitzki.
  Non-linear image registration.
  In O Scherzer, *Handbook of Mathematical Methods of Imaging*.
  Springer, 2014, to appear.
- A. Sotiras, C. Davatzikos, and N. Paragios.
  Deformable medical image registration: A survey.
  IEEE TMI, 32(7), 2013.

Mathematical Modelling

- Data model, images: $\mathcal{T}, \mathcal{R} \in \mathcal{C}_c^1(\mathbb{R}^d, G)$, here: $G = \mathbb{R}$
- Interpolation / approximation: $\mathcal{T}(x) = \text{model}(\mathtt{X}, \mathtt{T}, x)$
- Transformation model: $y : \mathbb{R}^d \to \mathbb{R}^d, \quad \mathcal{T}[y] := \mathcal{T} \circ y$

$$\mathcal{T}[y](x) := \mathcal{T}(y(x)) = \text{model}(\mathtt{X}, \mathtt{T}, y(x))$$

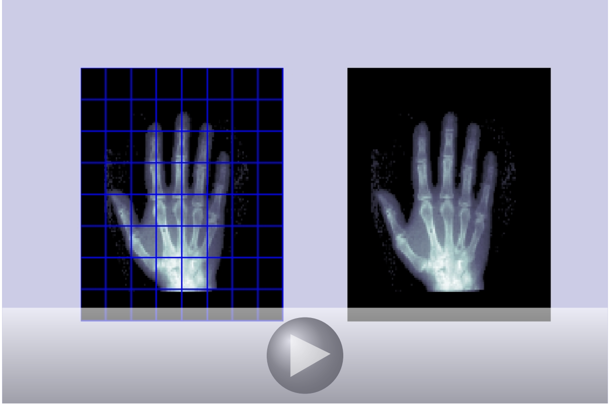
- Similarity of $\mathcal{T}[y]$ and $\mathcal{R}$: $\mathcal{D}[y] = \mathcal{D}[\mathcal{T}[y], \mathcal{R}]$
- Regularity of y: $\mathcal{S}[y]$
- Plausibility or constraints on y: $y \in \mathcal{A}$, admissible

Image Registration: Variational Formulation

$$\mathcal{J}[y] := \mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \alpha \mathcal{S}[y] \xrightarrow{y} \min, \qquad y \in \mathcal{A}$$

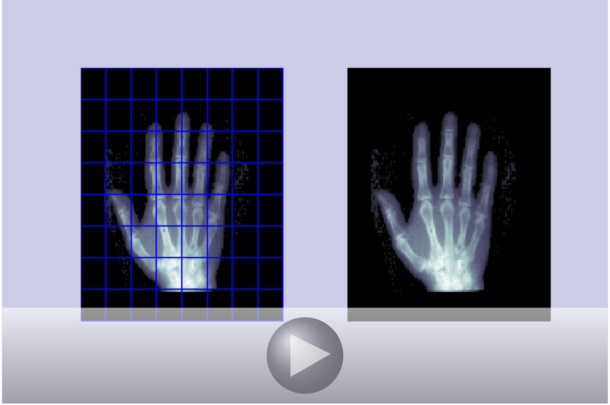
Transforming Images: Scaling

$$\mathcal{T}[y](x) = \mathcal{T}(y(x)) = \text{model}(\mathtt{X}, \mathtt{T}, y(x))$$



Transforming Images: Non-linear

$$\mathcal{T}[y](x) = \mathcal{T}(y(x)) = \mathrm{model}(\mathtt{X}, \mathtt{T}, y(x))$$



Optimize then Discretize

$$\mathcal{J}[y] := \mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \alpha\mathcal{S}[y] \xrightarrow{y} \min, \qquad y \in \mathcal{A}$$

- $\nabla\mathcal{J} = 0 \rightsquigarrow$ ELE, PDE

$$(y_t =) \quad f(y) + \alpha A y = 0$$

- Issues: root finding, no optimization; no solid stopping
- Discretization issues: symmetry, boundary conditions
- Intuitive (Thirion's Demons!):

  compute forces $f := f(y)$

  apply smoothing $y \leftarrow G_\alpha * f$, $G_\alpha \approx (\alpha A)^{-1}$

Discretize then Optimize

$$\mathcal{J}[y] := \mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \alpha \mathcal{S}[y] \stackrel{y}{\longrightarrow} \min, \qquad y \in \mathcal{A}$$

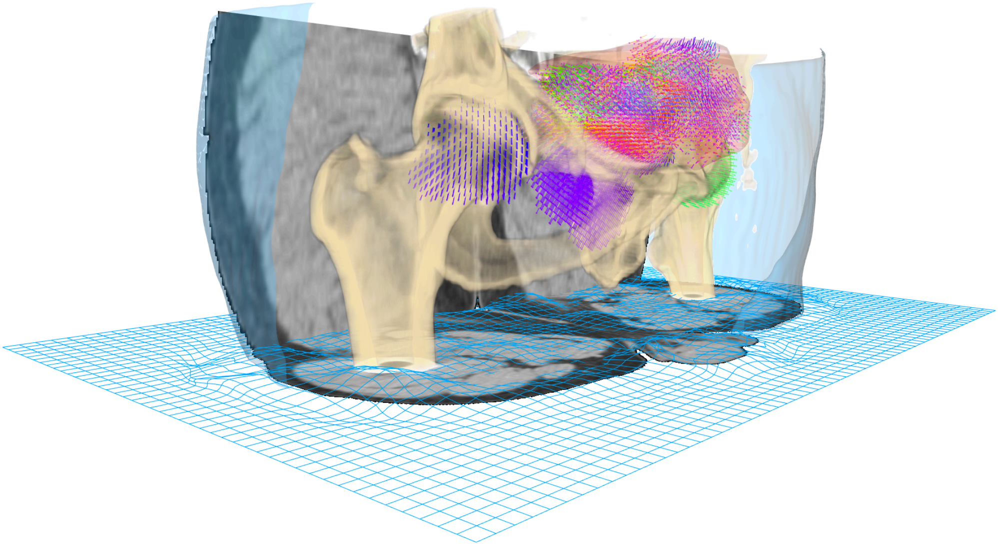
- discretizations $J^h, y^h, \quad h \longrightarrow 0$
- fixed h yields finite dimensional optimization problem

$$J(y^h) = D(y^h) + \alpha S(y^h) \stackrel{y^h}{\longrightarrow} \min, \qquad y^h \in \mathcal{A}^h$$

- efficient optimization schemes (Newton-type)
- proper stopping rules
- propagating y^h with respect to h

- Modersitzki, J: *FAIR – Flexible Algorithms for Image Registration*, SIAM, 2009

"Your x-ray showed a broken rib, but we fixed it with Photoshop."



Data Models and Images: $\mathcal{T}$ and $\mathcal{R}$

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

Outline

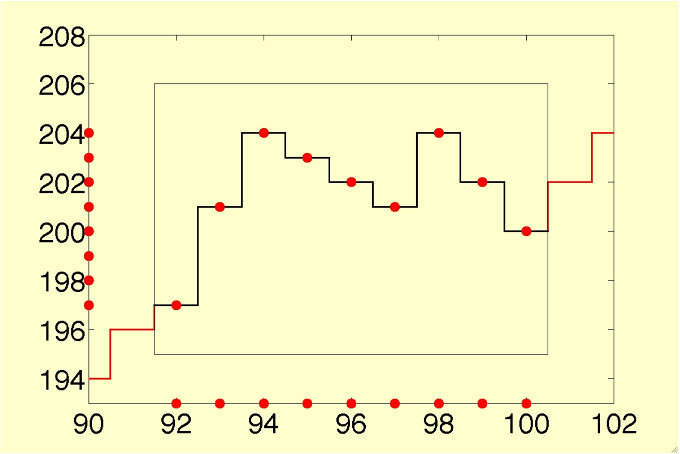
- General remarks, digital image processing
- Discrete data $\mapsto$ continuous
- Scale-space
- Multilevel
- Continuous model $\mapsto$ family of discretizations (grids, X^h, $h \to 0$)
- $\mathcal{T}[y]$: image transformations
- Derivatives ∂

Digital Image Processing

Digital Image Processing

$\mathcal{T} : \Omega \subset \mathbb{R}^d \to G \subset \mathbb{R}$,   image dimension $d = 2, 3, 4$

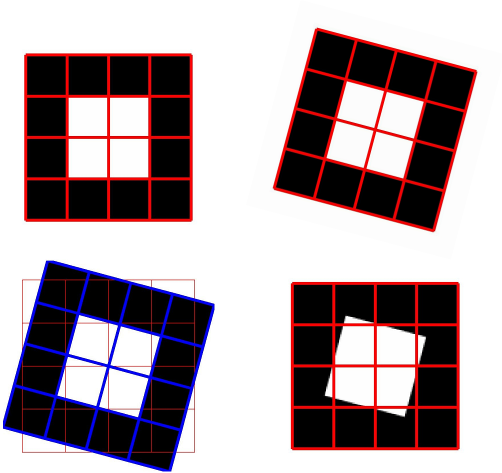
Images: a Closeup



images come discrete: $\mathtt{TD} : \underbrace{\Omega \cap \mathbb{Z}^d}_{=:\mathrm{XD},\ grid} \to G \cap \mathbb{Z}$

The world is not discrete

Rotation of a discrete 4-by-4 pixel image



pixels/voxels do not transform naturally – interpolation is required

D→C: discrete to continuous

Variational Approach for Image Registration

$$\mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

- Continuous models $\mathcal{R}, \mathcal{T}$ for reference and template:

  discrete data $\mathtt{X}, \mathtt{T} \rightsquigarrow \quad \mathcal{T}[x] = \mathrm{model}(\mathtt{X}, \mathtt{T}, x, \Theta)$

- Transformation $y : \mathbb{R}^d \to \mathbb{R}^d$

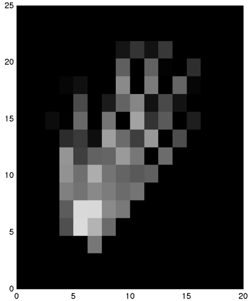
  $$\mathcal{T}[y](x) = \mathcal{T}[y(x)] = \mathrm{model}(\mathtt{X}, \mathtt{T}, y(x), \Theta)$$

Visualization and Discretization

MATLAB

```
f=@(x) sin(2*pi*x);
m=5; m=5001;
x=linspace(0,1,m);
plot(x,f(x),'b-');

setupHandData;
T=@(x) inter(dataT,omega,x);
m=[8,8]; m=[128,128];
x=getCellCenteredGrid(omega,m);
viewImagesc(T(x),omega,m);
```



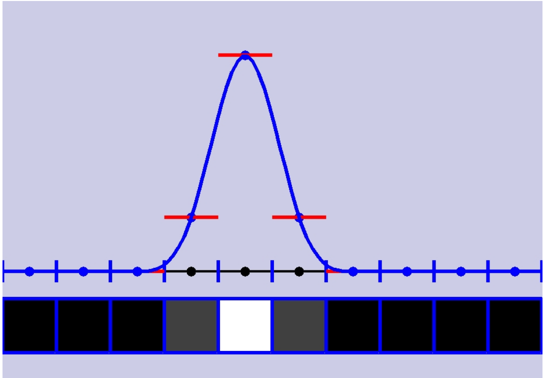
Integration: $h = 1/m$

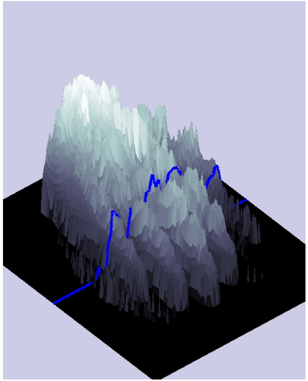
$$\int_0^1 f(x)\ dx = h \sum_{i=1}^{m} f(x_i) + \mathcal{O}(h^2), \qquad m = ???$$

coarse: h big, inexpensive, inaccurate

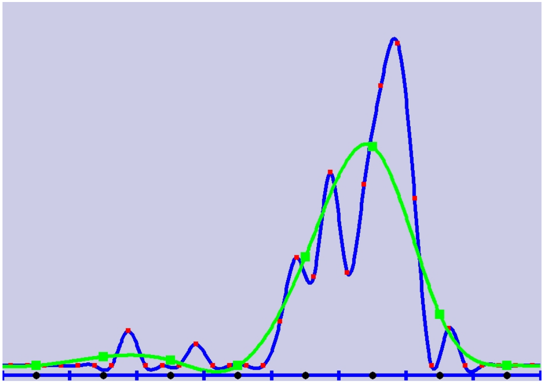
fine: h small, expensive, accurate

Interpolation





Multilevel

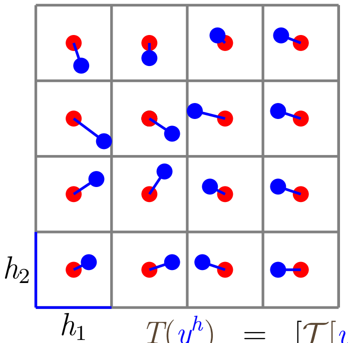


Sampling and Grids, Resolution

C$\rightarrow$D: continuous to discrete

Discrete Transformed Image

- Given: discrete data TD and XD and interpolation scheme
  $\mathcal{T}[x] = \text{model}(\texttt{XD}, \texttt{TD}, x)$, continuous representation
- Wanted: discrete transformed image, $T(y^h)$



- grid point, $i = (i_1, i_2)$
  $(x_i^{h,1}, x_i^{h,2}) := [(i_1 - \frac{1}{2})h_1, (i_2 - \frac{1}{2})h_2]$
- grid (collecting all grid-points)
  $x^h := [x^{h,1}, x^{h,2}] := [x_i^{h,1}, x_i^{h,2}]_i$
- transformed grid
  $y^h := [y^{h,1}, y^{h,2}] := [y_i^{h,1}, y_i^{h,2}]_i$

$T(y^h) = [\mathcal{T}[y_i^h]$, all $i]$ = discrete image

$\mathcal{T}[y] = [\mathcal{T}[y(x)]$, all $x]$ = continuous image

Derivatives

$\rightsquigarrow$ fast numerical optimization

Example: Derivatives for 2D Spline based $\mathcal{T}$

$$\begin{aligned}
\mathcal{T}[x] &= \sum c_{i,j}\, b_i(x^1)\, b_j(x^2) \\
\partial_1 \mathcal{T}[x] &= \sum c_{i,j}\, b_i'(x^1)\, b_j(x^2), \quad \text{analytic derivative!} \\
\partial_2 \mathcal{T}[x] &= \sum c_{i,j}\, b_i(x^1)\, b_j'(x^2), \quad \text{analytic derivative!}
\end{aligned}$$

y collection of n points in d dimensional space (here, $d = 2$)

$$y = \begin{pmatrix} (y_1^1, y_1^2) \\ \vdots \\ (y_n^1, y_n^2) \end{pmatrix}$$

$$T(y) = \underbrace{\begin{pmatrix} \mathcal{T}[y_1^1, y_1^2] \\ \vdots \\ \mathcal{T}[y_n^1, y_n^2] \end{pmatrix}}_{\in \mathbb{R}^n}, \quad dT = \underbrace{\begin{pmatrix} \partial_1 \mathcal{T}[y_1] & & & \partial_2 \mathcal{T}[y_1] & & \\ & \ddots & & & \ddots & \\ & & \partial_1 \mathcal{T}[y_n] & & & \partial_2 \mathcal{T}[y_n] \end{pmatrix}}_{\in \mathbb{R}^{n,dn}, \text{ sparse!}}$$

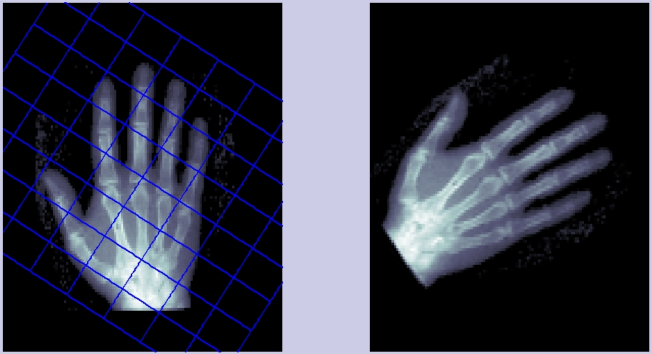
dT is an analytic derivative, don't mess with gradients!

Transforming Images

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

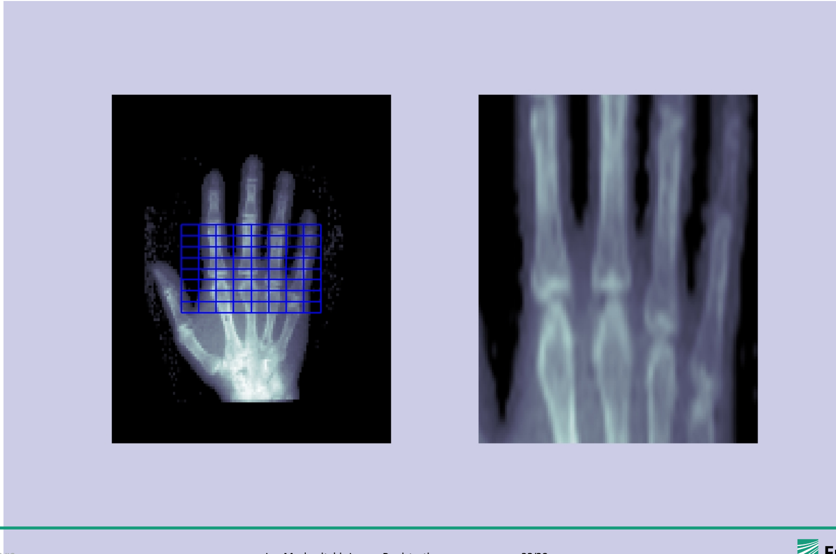
Transforming Images, rotation

$$\mathcal{T}[y](x) = \mathcal{T}[y(x)] = \text{model}(\texttt{X}, \texttt{T}, y(x))$$



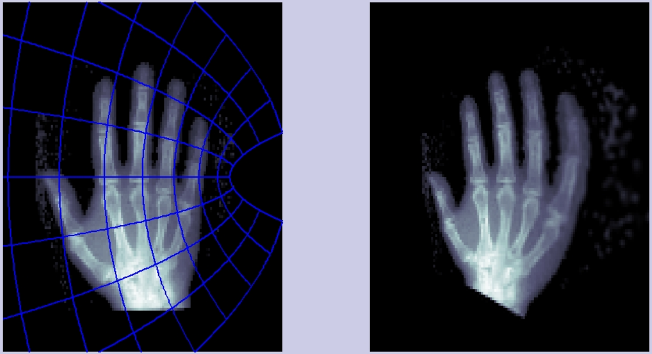
Transforming Images, scale

$$\mathcal{T}[y](x) = \mathcal{T}[y(x)] = \text{model}(\mathtt{X}, \mathtt{T}, y(x))$$



Transforming Images, non-linear

$$\mathcal{T}[y](x) = \mathcal{T}[y(x)] = \text{model}(\texttt{X}, \texttt{T}, y(x))$$



Transformation Models

Parametric transformations

- Finite-dimensional and linear space $\mathcal{Q}$, $\quad y \in \mathcal{Q}$

$$y(x) = \sum_{j=1}^{m} w_j \, q_j(x) = Q(x) \; w, \qquad \text{or: } y(w,x) = Q(x) \; P(w)$$

Non-Parametric transformations

- regularize y by $\mathcal{S}[y]$

Example: Affine Linear Transformations

$$y(w,x) = \sum_{j=1}^m w_j q_j(x) = Q(x) w, \qquad \text{or: } y(w,x) = Q(x)\; P(w)$$

2D affine linear, six degrees of freedom: $w = (w_1^1, w_2^1, w_3^1, w_1^2, w_2^2, w_3^2)^\top$

$$\begin{aligned} y(w,x) &= \begin{pmatrix} w_1^1 & w_2^1 \\ w_1^2 & w_2^2 \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} + \begin{pmatrix} w_3^1 \\ w_3^2 \end{pmatrix} \\ y^\ell(w,x) &= w_1^\ell\; x^1 + w_2^\ell\; x^2 + w_3^\ell \cdot 1 \quad = [x^1, x^2, 1]\, w^\ell, \quad \ell = 1,2 \end{aligned}$$

$$y(w,x) = \begin{pmatrix} x^1 & x^2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x^1 & x^2 & 1 \end{pmatrix} w = Q(x)\; w$$

Interpolation Toolbox

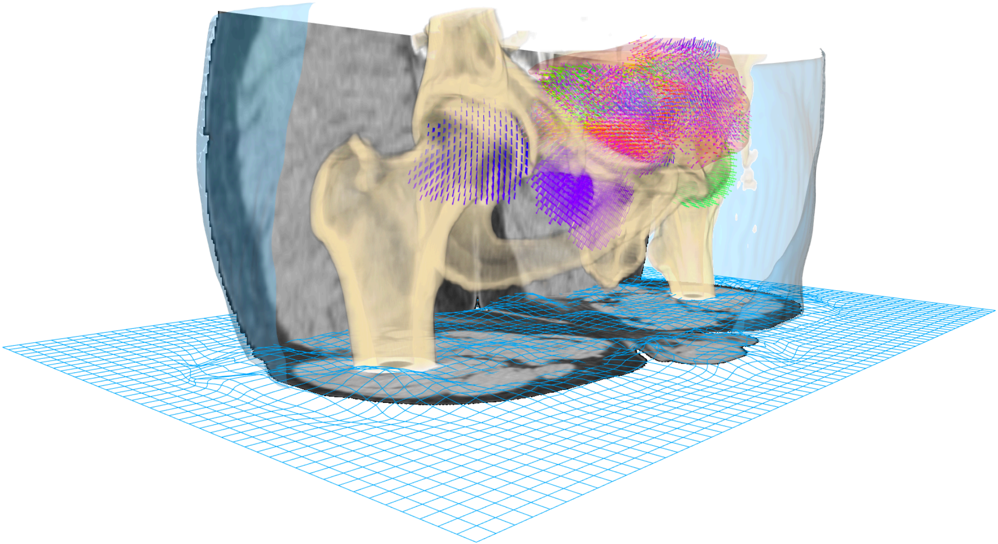
- Continuous models $\mathcal{R}, \mathcal{T}$ for reference and template required interpolation / approximation / scale-space

$$\mathcal{T}[x] := \mathrm{model}(\mathtt{X}, \mathtt{T}, x, \Theta)$$

- Differentiability: analytic derivatives a.e.
- Multi-resolution framework
- Transformed image (Eulerian framework)

$$\mathcal{T}[y](x) := \mathcal{T}[y(x)] = \mathrm{model}(\mathtt{X}, \mathtt{T}, y(x), \Theta)$$

- Numerics: discretization x^h of Ω, T^h of $\mathcal{T}$, R^h of $\mathcal{R}$
- Multilevel ! (discussed later)



Distance Measures

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

Example

R

0	0	0	0	0	0
0	0	0	0	0	0
0	0	4	4	0	0
0	0	4	4	0	0
0	0	0	0	0	0
0	0	0	0	0	0

T

0	1	2	3	2	1
0	1	2	3	2	1
0	1	2	3	2	1
0	1	2	3	2	1
0	1	2	3	2	1
0	1	2	3	2	1

$(T_i - R_i)^2$

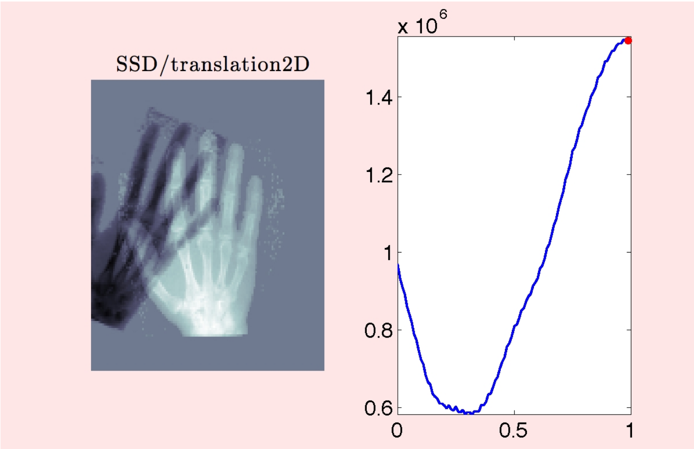
0	1	4	9	4	1
0	1	4	9	4	1
0	1	4	1	4	1
0	1	4	1	4	1
0	1	4	9	4	1
0	1	4	9	4	1

$$\mathcal{D}[\mathcal{T},\mathcal{R}] = \frac{1}{2}\int_{\Omega} (\mathcal{T}[x] - \mathcal{R}(x))^2 \; dx \approx \frac{h}{2} \sum_i (T_i - R_i)^2$$

Example: SSD versus shifts, linear $\mathcal{T}[y]$

Example: $y(x) = (x_1 + w, x_2)$, w shift parameter

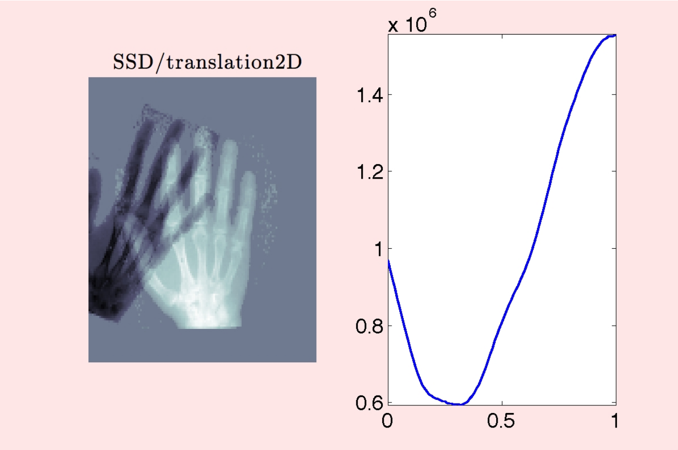
$|\mathcal{T}[w] - \mathcal{R}|$ $\mathcal{D}^{\text{SSD}}[\mathcal{T}[w], \mathcal{R}]$ versus w



Example: SSD versus shifts, B-spline $\mathcal{T}[y]$

Example: $y(x) = (x_1 + w, x_2)$, $\quad w$ shift parameter

$|\mathcal{T}[w] - \mathcal{R}|$ $\qquad$ $\mathcal{D}^{\mathrm{SSD}}[\mathcal{T}[w], \mathcal{R}]$ versus w

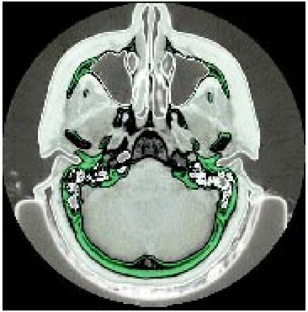


Sum of Squared Differences

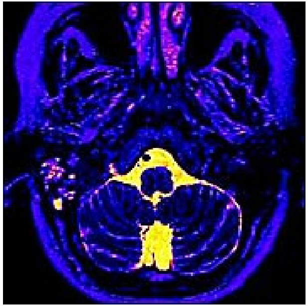
- Simple
- Robust, least squares measure
- Intuitive
- Fast to compute, $\mathcal{O}(n)$

- Only for images of the same modality
- Integral measure, no point-to-point correspondence

Multi-Modal Images



?



Motivation

Goal: simple, intensity independent distance measure

Observation: $\mathcal{D} = \mathcal{D}[\mathcal{T}[y], \mathcal{R}]$

$$\underbrace{d_y \mathcal{D}}_{\text{force field}} = \underbrace{d_{\mathcal{T}} \mathcal{D}}_{\text{weights}} \cdot \underbrace{\nabla \mathcal{T}^{\top}}_{\text{edge image}}$$

Idea: use the gradient field $\nabla \mathcal{T}$ directly

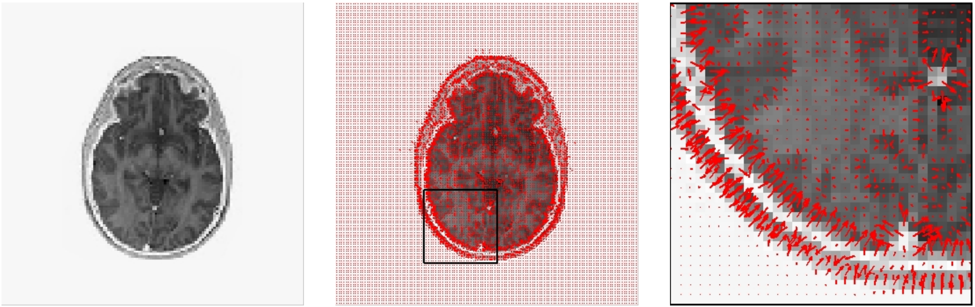
Motivation

But: ignore intensities $\rightsquigarrow$ $\vec{n} = \dfrac{\nabla \mathcal{T}}{\|\nabla \mathcal{T}\|}$

Regularize: $\vec{n}_\eta = \dfrac{\nabla \mathcal{T}}{\sqrt{\|\nabla \mathcal{T}\|^2 + \eta^2}}$

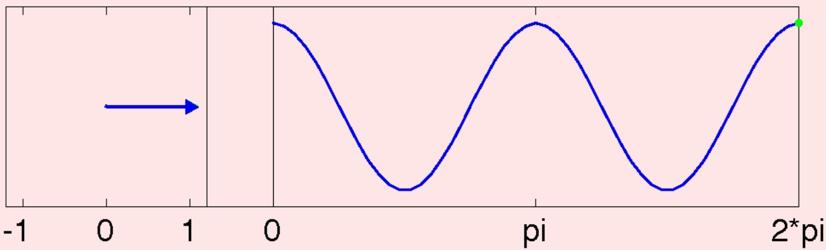
edge-parameter η: differentiates edges and noise

Example: Normalized Gradient Field



Gradient based distance measure

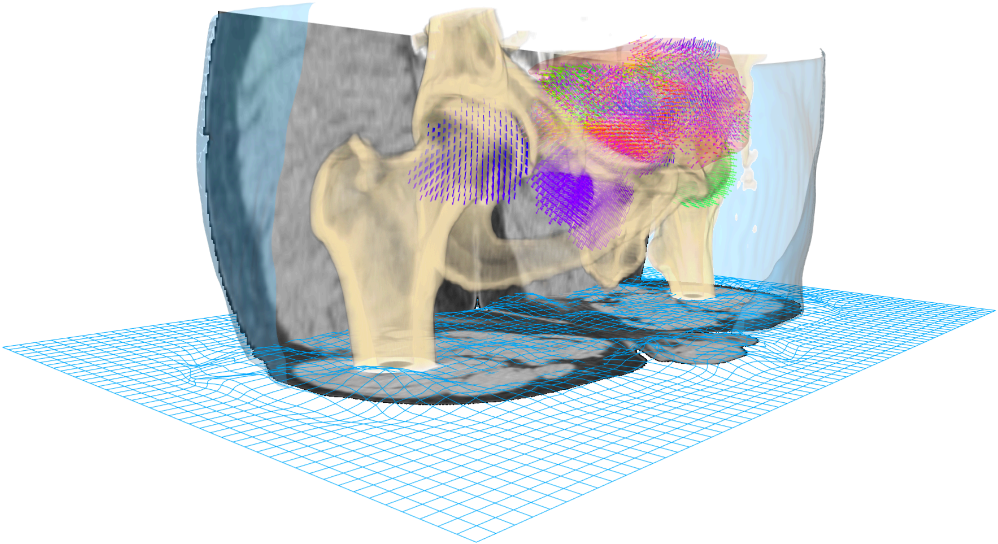
pointwise linear dependency of $\vec{n}^{\mathcal{T}}$ and $\vec{n}^{\mathcal{R}}$: $\cos \angle(\vec{n}^{\mathcal{T}}, \vec{n}^{\mathcal{R}})^2$



$$\mathcal{D}^{\mathrm{NGF}}[\mathcal{T},\mathcal{R}] = \int_\Omega 1 - \left((\vec{n}^{\mathcal{T}})^\top \vec{n}^{\mathcal{R}} \right)^2 dx$$

Normalized Gradient Fields

- Two images are considered to be similar,
  if intensity changes occur at the same locations
- Focus on edges not on intensities
- Intuitive and interpretable,
  distances relate to spatial positions
- No tuning parameters
- Accessible for fast (derivative based) optimization schemes
- Accessible for multi-resolution
- Easy and straightforward to implement



Parametric Image Registration

$$\mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

Parametric Image Registration

Reduced Model

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] \stackrel{y}{=} \min \quad \text{subject to} \quad y \in \mathcal{Q}$$

- Distance measure $\mathcal{D}$ (e.g., SSD)
- Finite-dimensional and linear space $\mathcal{Q}$

$$y(w,x) = \sum_{j=1}^{m} w_j q_j(x) = Q(x)\; w, \qquad \text{or: } y(w,x) = Q(x)\; P(w)$$

Fraunhofer MEVIS

Optimization

Parametric Registration

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] \stackrel{y}{=} \min \quad \textbf{s.t.} \quad y \in \mathcal{Q} = \{Q(x)\ w,\ w \in \mathbb{R}^p\}$$

$$\begin{aligned} \mathcal{D}[\mathcal{T}[y],\mathcal{R}] &\approx \tfrac{h}{2}\,\|T(y^h)-R\|^2_{\mathbb{R}^n} &=:\ & D(y^h) \\ y = Q(x)\ w &\approx Q(x^h)w = Q^h w &=:\ & y^h \end{aligned}$$

Discretized Parametric Registration

$$D(w) = D(Q^h w) = \tfrac{h}{2}\,\|T(Q^h w)-R\|^2_{\mathbb{R}^n} \stackrel{w}{=} \min$$

Optimization techniques:

- Steepest Descent, Gauß-Newton, Levenberg-Marquardt, BFGS, …

Gauß-Newton scheme

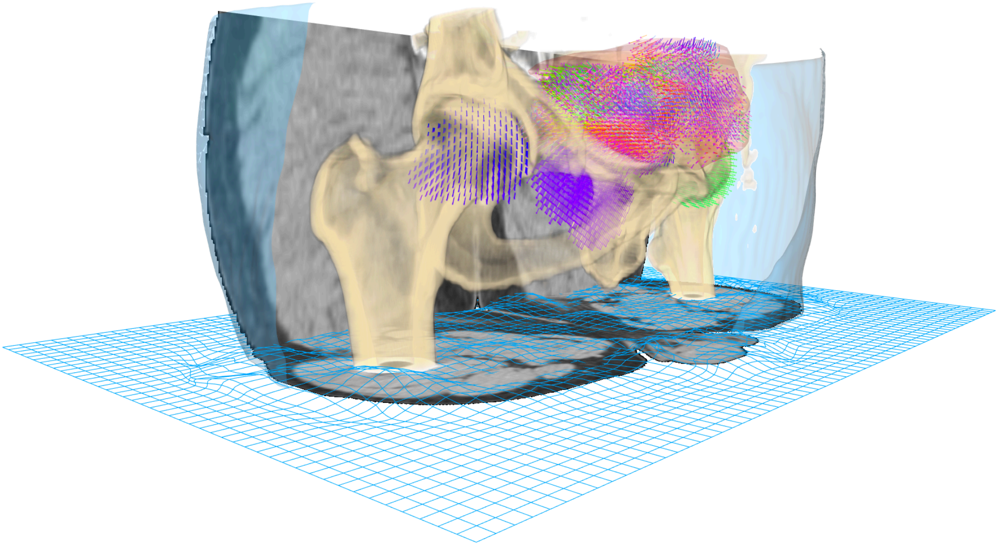
- Goal: $D(r(w)) \stackrel{w}{=} \min$
- Quadratic model: $H = dr^\top d^2 D\, dr + \text{"}\, dD\, d^2 r\text{"}$

- Gauß-Newton: linearize the inner function

$$D(r(w+u)) \approx D(r + dr\ u) \stackrel{u}{=} \min$$

- Necessary condition for minimizer: $d_w D + Hu \stackrel{!}{=} 0$

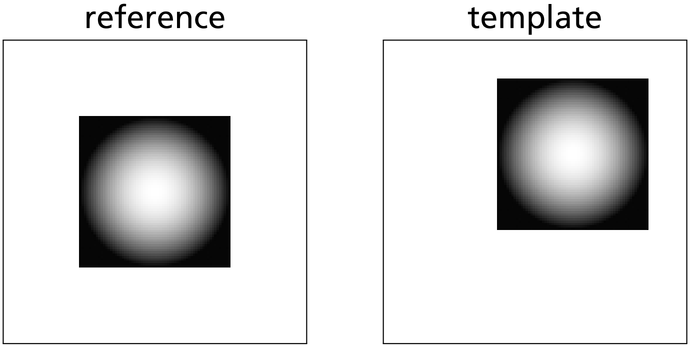
$$H = d_u^2 D(r + dr\ u) = dr^\top d^2 D\, dr$$



Regularization

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

Ill-posedness of Parametric Registration



How to match reference and template using only rigid transformations?

Implicit versus Explicit Regularization ...

Registration is ill-posed $\rightsquigarrow$ requires regularization

- Parametric Registration
  - restriction to (low-dimensional) space (rigid, affine linear, spline,...)
  - regularized by properties of the space (implicit)
  - not physical or model based
- Non-parametric Registration
  - regularization by adding penalty or likelihood (explicit)
  - allows for a physical model
  - $\rightsquigarrow$ y is no longer parameterizable

… Implicit versus Explicit Regularization

registration is ill-posed $\rightsquigarrow$ requires regularization

Parametric Registration

$\mathcal{D}[\mathcal{R},\mathcal{T};y] \overset{y}{=} \min$ subject to subject to $y \in \mathcal{Q} = \{Qw,\ w \in \mathbb{R}^m\}$

Regularized Parametric Registration

$\mathcal{D}[\mathcal{R},\mathcal{T};y] + \alpha\mathcal{S}[y] \overset{y}{=} \min$ subject to $y \in \mathcal{Q} = \{Qw,\ w \in \mathbb{R}^m\}$

Non-parametric Registration $\mathcal{D}[\mathcal{R},\mathcal{T};y] + \mathcal{S}[y] \overset{y}{=} \min$

Overview

- Registration is ill-posed $\rightsquigarrow$ requires regularization
- Regularizer controls reasonability of transformation
- Application conform regularization
- Enabling physical models
  (linear elasticity, fluid flow, hyperelasticity, ...)
- $\rightsquigarrow$ high dimensional optimization problems

Regularizer

$$\mathcal{S}[y] = \alpha\, \mathcal{S}^{\text{ext}}[y - y_{\text{reg}}]$$

Regularizer $\mathcal{S}$

transformation / displacement, $y(x) = x + u(x,t)$

- "elastic" $\mathcal{S}^{\text{elas}}[u] =$ elastic potential of u
- "fluid" $\mathcal{S}^{\text{fluid}}[u] =$ elastic potential of $\partial_t u$
- "diffusion" $\mathcal{S}^{\text{diff}}[u] = \frac{1}{2}\sum_{\ell=1}^{d}\int_\Omega \|\nabla u_\ell\|_{\mathbb{R}^2}^2 \; dx$
- "curvature" $\mathcal{S}^{\text{curv}}[u] = \frac{1}{2}\sum_{\ell=1}^{d}\int_\Omega (\Delta u_\ell)^2 \; dx$
- "hyperelastic" $\mathcal{S}^{\text{hyper}}[\nabla y, \operatorname{cof} \nabla y, \det \nabla y] = \cdots$
- ...

Elastic Registration

transformation / displacement, $y(x) = x + u(x)$

$$\mathcal{S}^{\text{elas}}[u] = \text{elastic potential of } u$$
$$= \int_\Omega \frac{\lambda + \mu}{2} \| \nabla \cdot u\|^2 + \frac{\mu}{2} \sum_{i=1}^{d} \|\nabla u_i\|^2 \; dx$$

image painted on a rubber sheet

- C. Broit.
  Optimal Registration of Deformed Images.
  PhD thesis, University of Pensylvania, 1981.
- Bajcsy & Kovačič 1986, Christensen 1994, Bro-Nielsen 1996, Gee et al. 1997, Fischer & M. 1999, Rumpf et al. 2002, …

Diffusion Registration

transformation / displacement, $y(x) = x + u(x)$

$$\begin{aligned} \mathcal{S}^{\text{diff}}[u] &= \text{oszillations of } u \\ &= \tfrac{1}{2} \sum_{\ell=1}^{d} \int_\Omega \|\nabla u_\ell\|_{\mathbb{R}^d}^2 \, dx \end{aligned}$$

heat equation

- B. Fischer and J. Modersitzki.
  Fast diffusion registration.
  AMS Contemporary Mathematics, Inverse Problems, Image Analysis, and Medical Imaging, 313:117–129, 2002.
- Horn & Schunck 1981, Thirion 1996, Droske, Rumpf & Schaller 2003, ...

Curvature Registration

transformation / displacement, $y(x) = x + u(x)$

$$\mathcal{S}^{\text{curv}}[u] = \text{oszillations of } u$$
$$= \frac{1}{2} \sum_{\ell=1}^{d} \int_{\Omega} \|\Delta u_\ell\|_{\mathbb{R}^d}^2 \, dx$$

bi-harmonic operator

B. Fischer and J. Modersitzki.
Curvature based image registration.
J. of Mathematical Imaging and Vision, 18(1):81–85, 2003.

Stefan Henn.
A multigrid method for a fourth-order diffusion equation with application to image processing.
SIAM J. Sci. Comput., 2005.

Large Deformations

$$\mathcal{S}[\nabla y, \operatorname{cof} \nabla y, \det \nabla y] = \int \|\nabla y\|^2 + \phi(\|\operatorname{cof} \nabla y\|^2_{\text{Fro}}) + \psi(\det \nabla y) \; dx$$

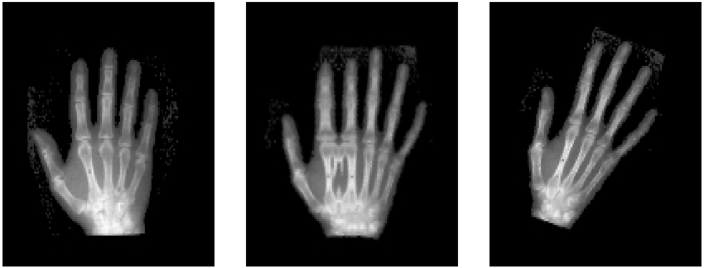
S. Darkner et al.
Efficient Hyperelastic Regularization for Registration
Scandinavian Conference on Image Analysis, Ystad, Sweden, 2011.

M. Burger, J. Modersitzki, L. Ruthotto
A hyperelastic regularization energy for image registration.
submitted to SIAM SISC, 2012.

Regularizer

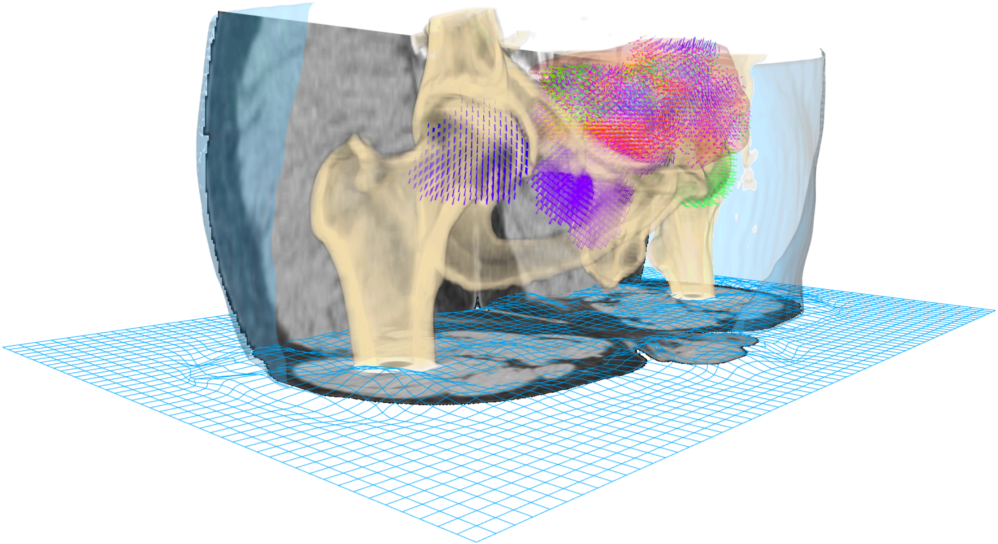
$$\mathcal{S}[y] = \alpha\, \mathcal{S}^{\mathrm{ext}}[y - y_{\mathrm{reg}}]$$

Does regularization matter? It does!



transformed template $\mathcal{T}[y]$

- MLIR, SSD, non-linear elasticity, spline interpolation



Numerical Methods for IR

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

Discretize $\rightarrow$ Optimize: Concept

Discretization $\rightsquigarrow$ finite dimensional problem: $y^h \approx y(x^h)$

$$D(y^h) + S[y^h] \xrightarrow{y^h} \min, \quad y^h \in \mathbb{R}^n, \qquad h \longrightarrow 0$$

- efficient optimization schemes (Newton-type)
- linear systems of type $H\,\delta_y = -\mathrm{rhs}$,

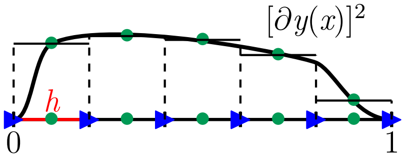
$$H = M + B^\top B, \quad M \approx D_{yy}, \quad \mathrm{rhs} = D_y + (B^\top B) y^h$$

- efficient multigrid solver for linear systems
- large steps
- discretization not straightforward (multigrid)
- all parts have to be differentiable (data model)

Example: 1D, SSD, and diffusion

$$
\begin{aligned}
\mathcal{J}[y] &= \mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min \\
&\stackrel{\text{e.g.}}{=} \tfrac{1}{2}\int_0^1 [\mathcal{T}[y(x)] - \mathcal{R}(x)]^2 \, dx + \tfrac{1}{2}\int_0^1 [\partial y(x)]^2 \, dx \\
&\stackrel{\text{dis}}{\approx} \tfrac{1}{2}\int_0^1 [\mathcal{T}[y(x)] - \mathcal{R}(x)]^2 \, dx + \tfrac{1}{2} h \| B^h y^h \|_{\mathbb{R}^n}^2 \\
&\stackrel{\text{dis}}{\approx} \tfrac{1}{2} h \| T(y^h) - R \|_{\mathbb{R}^n}^2 + \tfrac{1}{2} h \| B^h y^h \|_{\mathbb{R}^n}^2 \\
&=: J(y^h)
\end{aligned}
$$

1D discretization, regularizer



midpoint rule for $\int_0^1 [\partial y(x)]^2 \; dx$

$\partial y(x_{j+0.5}) \approx (y^h_{j+1} - y^h_j)/h$

short differences!

- y^h is on staggered grid, ∂y is on cell-centered grid $x^h_\bullet$
- $(\partial y)^h = B^h y^h$,
- midpoint rule

$$\int_0^1 [\partial y(x)]^2 \; dx \approx h\sum_{j=1}^n [\partial y(x_{j-0.5})]^2$$

$$\approx h\sum_{j=1}^n [B^h y^h]_j^2 = h\|B^h y^h\|^2_{\mathbb{R}^n}$$

Discretization of

L_2-norm Based Regularizer

L_2-norm Based Regularizer

$$\mathcal{S}[y] = 0.5\|\mathcal{B}[y]\|^2_{L_2(\Omega)} = 0.5 \int_\Omega \mathcal{B}[y]^\top \mathcal{B}[y] \; dx$$

Example

- $d = 1$, $\mathcal{B}[y] = y'$, $\mathcal{S}[y] = \int_0^\omega (y')^2 \; dx$
- diffusion, $d = 2$: $\mathcal{B}[y] = [\nabla y^1; \nabla y^2]$,

$$\mathcal{S}[y] = \int_\Omega (\partial_1 y^1)^2 + (\partial_2 y^1)^2 + (\partial_1 y^2)^2 + (\partial_2 y^2)^2 \; dx$$

- elastic, $d = 2$: $\mathcal{B}[y] = [\nabla y^1; \nabla y^2; \; \nabla \cdot y]$

Discretized Regularizer

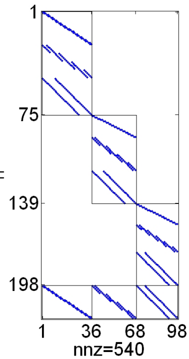
- $\mathcal{S}[y] = \frac{1}{2} \int_\Omega \mathcal{B}[y]^\top \mathcal{B}[y] \, dx$
  bi-linear form, differential operator $\mathcal{B}$
- $\mathcal{B} \approx (\nabla, \nabla\times, \nabla\cdot) \quad \rightsquigarrow \quad B \approx (\nabla^h, \nabla^h\times, \nabla^h\cdot)$
- $\int_\Omega \quad \rightsquigarrow \quad h\sum_i$
- important for multigrid solver: short differences!
  $\rightsquigarrow$ staggered grids for y^h
- $S[y^h] = \frac{1}{2} \left\| B\, y^h \right\|^2_{\mathbb{R}^n}$, $\qquad dS = B^\top B\, y^h$, $\qquad d^2 S = B^\top B$
- diffusion regularizer: $B^\top B = -\Delta^h$
- elastic regularizer: $\quad B^\top B = -\mu\Delta^h - (\mu + \lambda)\nabla^h \nabla^h\cdot$

Remarks on Discrete Operator B

- Big
- Very sparse
- Structure
- Not needed: matrix-free code for

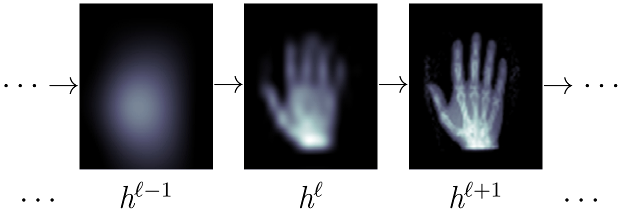
  By, $B^\top z$, $(B^\top B)y = \text{rhs}$

$B =$



Multilevel Strategie

Multilevel



for $\ell = 1 : \ell_{\max}$ **do**

transfer images to level ℓ

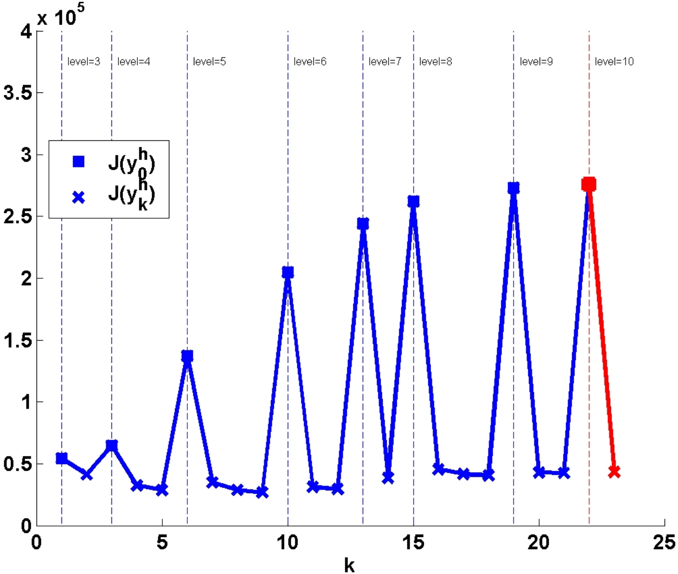
approximately solve problem for y

prolongating y to finer level $\rightsquigarrow$ perfect starting point

end for


J^{fine}
J^{coarse}
$J^{coarsest}$

Example: Multilevel Iteration History



Advantages of Multilevel Strategy

- Regularization
- Focus on essential minima
- Creates extraordinary starting value
- Reduces computation time

- ?

Summary of Numerical Optimization

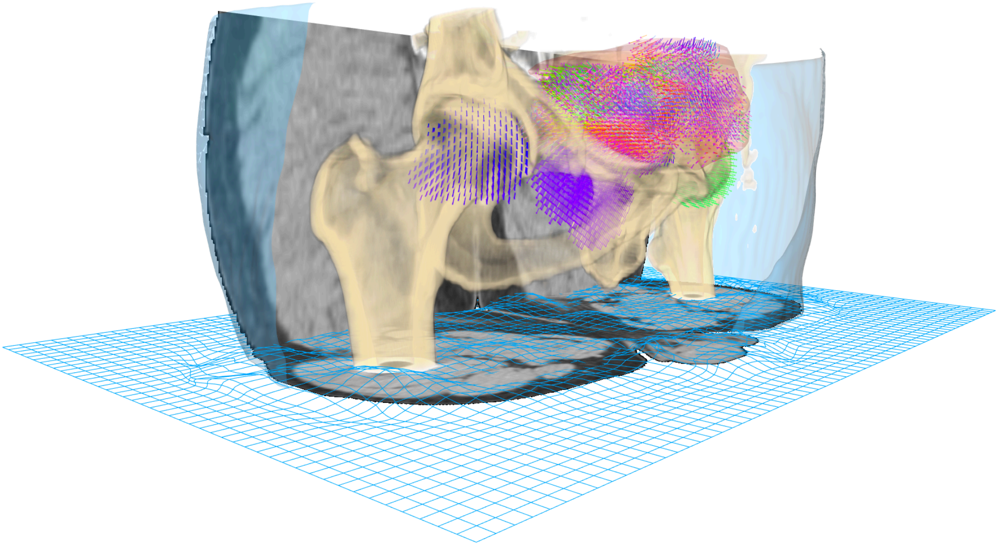
- Optimize $\leftrightarrow$ Discretize

Advantages of Discretize $\to$ Optimize

- efficient numerical optimization (Gauß-Newton, ℓBFGS, Trust-Region)
- appropriate stopping criteria
- efficient line search strategies

Core: solve $H\delta_y = -\mathrm{rhs}$

- preconditioned conjugate gradient with multigrid as preconditioner
- requires differentiable modules
- requires multigrid amenable discretization



Hands-On

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y] \xrightarrow{y} \min$$

Hands on Session

- Small groups $\sim$ 5-6 persons
- Matlab-Demonstration using FAIR

- Transformation
- Parametric Image Registration
- Non-parametric Image Registration

- **J. Modersitzki**, FAIR Flexible Algorithms for Image Registration, SIAM, 2009,
- See also `http://www.siam.org/books/fa06`
  (for software: copyright form needs to be signed)

- After the break!